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Faded-example as a Tool to Acquire and Automate Mathematics Knowledge

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Abstract. Students themselves accomplish Knowledge acquisition and automation. The teacher plays a role as the facilitator by creating mathematics tasks that assist students in building knowledge efficiently and effectively. Cognitive load caused by learning material presented by teachers should be considered as a critical factor. While theintrinsic cognitive load is related to the degree of complexity of the material learning ones can handle, theextraneous cognitive load is directly caused by how the material is presented. Strategies to present a learning material in computational learning domains like mathematics are anamely worked example (fully-guided task) or problem-solving (discovery task with no guidance). According to the empirical evidence, learning based on problem-solving may cause high-extraneous cognitive load for students who have limited prior knowledge, conversely learn based on worked example may cause high-extraneous cognitive load for students who have mastered the knowledge base. An alternative is a fadedexample consisting of thepartly-completed task. Learning from faded-example can facilitate students who already acquire some knowledge about the to-be-learnedmaterial but still need more practice to automate the knowledge further. This instructional strategy provides a smooth transition from a fully-guided into an independent problem solver. Designs of faded examples for learning trigonometry are discussed.

1. Introduction

Learning is a cognitive process to construct knowledge in working memory and store it permanently in long-term memory [1]. These two kinds of memory systems are the major components of our cognitive architecture. Having a well-constructed knowledge means acquiring and organising knowledge meaningfully thus enables students to understand deeply how the knowledge is applied. Moreover, learning will be enhanced when this knowledge can be automated. It is suggested that knowledge automation occurs by deliberate practice (i.e., planned, regular, aim into specificity) [2]. Knowledge automation benefits students to solve problems faster, perform less error and advance learning. Accordingly, it is important to note that facilitating learning is not only to assist students acquiring knowledge but also automating knowledge.

Mathematics is a computational learning domain that has a well-structured knowledge building. It consists of operations and algorithms on how to solve problems. Indeed, it has been recommended that mathematics should be learned by problem-solving[3]. Problem-solving is an activity to solve complex problems. Based on the given complex problem, students attempt to understand mathematical concepts underpinning the problem, and at the same time, apply the algorithms or procedures of solving it. However, solving complex problems are not always easy. Different students may treat complexity differently depending on many factors, such as prior knowledge or motivation. Hence they require different instruction [4]. Consequently, teachers should employ strategies of mathematics instructions by which students can master mathematics knowledge underpinning the problems solving.

Cognitive load theory is an instructional design theory that draws its principles based on empirical evidence [1]. One of the principles is that learning occurs effectively when cognitive load is at amanageable level. Cognitive load results when we are organising learning material in our working memory. Two types of cognitive load have been defined, intrinsic and extraneous cognitive load [5, 6]. While theintrinsic cognitive load is caused by the degree of interactivity of the elements of the learning material the student can organise, theextraneous cognitive load is triggered by the presentation of the learning material. Working memory resource has to deal with these cognitive loads simultaneously. The germane cognitive load is a cognitive load that is devotedto acquiring and constructing knowledge [6]. Its load depends on how much working memory manages intrinsic and extraneous cognitive load. Extraneous cognitive load tends to hinder learning, thus minimising extraneous cognitive load is necessary to increase the chance of germane cognitive load.

The complexity of a learning material is about student's prior knowledge. If the pre-requisite knowledge exists in student's long-term memory and can be retrieved automatically, intrinsic cognitive load in working memory may be lower. It occurs because the existing knowledge guides students to recognise, identify and organise the to-be-learned material. However, if the pre-requisite knowledge does not exist, then it is unlikely to recognise the presented learning material. Knowledge of what students have would assist teachers to present sufficient task for students and thus learning is facilitated.

About extraneous cognitive load, it is suggested that learning material is presented thoughtfully. For instance, it provides easy-to-follow guidance or worked-example if the material is presented to novice students, or it has sufficient problem-solving task if presented to master students. As previously noted, which instructional strategy to apply should refer to student's level of prior knowledge. Based on cognitive load theory, aninstruction that uses problem-solving as the major activity may cause high-extraneous cognitive load if presented to students who are alack of prior knowledge. On the other hand, an instruction based on worked example may cause high-extraneous cognitive load for students who have advanced prior knowledge [7]. In fact, there are students who have learned some knowledge but limited ability to transfer it into problem-solving. An alternative instruction for these students may be an instruction based on faded-examples. Faded-examples consist of partly-completed problem-solving. It may be applicable for students who have some of therequired knowledge bases with limited use. This paper discusses further this faded-example strategy for mathematics learning, as follows.

2. Worked-Example

A conventional teaching method, commonly a teacher starts with theintroduction of the topic, short explanation, then one or two worked examples followed by practice a set of problem-solving. Worked example is often used to provide adescription how to step-by-step solve a mathematics problem [8]. Atkinson, Derry [9] suggested that worked examples should demonstrate how a mathematician expert solves the problem, not simply a problem solution but a solution that assist students to understand and is powerful to imitate.

A worked example based instruction facilitates students to learn from a set of worked examples. This instruction may belong to a student-centered learning paradigm. In the classroom, teachers who apply this instruction begin thelesson with some introductory knowledge base for the to-be-learned material and explanation of what learn by illustrating or discussing a novel mathematics problem they are about to learn. It may challenge yet motivate students. The main learning activity is followed by giving a set of worked example instruction for students to learn by themselves; hence students have their phase to acquire new knowledge while completing the instructional task. After this learning phase, students are given theopportunity to confirm their newly constructed knowledge through thepresentation, correctness checking or further problem-solving elaboration with the other students or the teacher.

Worked example effect occurs when an instruction based on worked examples is proved to be more effective to facilitate learning compared to another minimally guided based instruction, such as problem-based learning. An explicit guidance decreases extraneous cognitive load by avoiding anirrelevant search of meaning during learning. Worked examples eliminate the use of means-ends analysis by asking students to learn a solution to a novel problem solving, rather than asking students to discover by themselves. In turn, knowledge acquired from worked examples will play a role as aknowledge base when given similar problem-solving. The existence of knowledge base is essential for learning and hence working memory is devoted to knowledge acquisition and automation.

Atkinson, Derry [9]summarised that research into the use of worked examples for learning new material had been carried out for more than six decades using various subject matters. A few evidence for the worked example effect in mathematicswas provided by Zhu and Simon [10] in their longitudinal study using a 3-year curriculum in algebra and geometry in a Chinese middle school, where they found that students studying the worked examples could complete the three-year course in only two years. Research by Tarmizi and Sweller [11] using geometry also found the worked example effect, once split-attention was avoided Further evidence of the worked example effect were provided by Chi, Bassok [12] using mathematics for physics, and Ward and Sweller [13] using geometric optics and kinematics.

2.1. Principles of creating worked examples

Effectively worked examples can be created based on principles developed by the cognitive load theory. These principles are usually explainedregarding whether the modified format of worked example based instruction decreases or increases either extraneous or intrinsic cognitive loads and thus manages germane cognitive load. These principles guide teachers how to create the format of worked examples, how many examples should be presented, its variation and how to ensure that students attend the task fully.

The first principle is to reduce theextraneous load. It canbe done by arranging the presented information in such a way connecting elements or relevant information can be simultaneously grasped. Evidence from the split attention or spatial–contiguity effects supports this principle [11, 14]. When learners are imposed to do anunnecessary search and connect elements of information to understand the meaning of presented information, this will cause high extraneous cognitive load. It happens when relevant information that must be considered simultaneously are spatially separated or successively delivered. Accordingly, providing a clear instruction will surely help students getting in the learning material. Moreover, redundant information must be omitted [15]. Redundancy effect may be caused by presenting unnecessary information, adding extra information to already self-contained information, using multiple formats for the same information or repeating similar information concurrently. Such information may distract students from understanding the learning material, and hence cause heavy extraneous cognitive load.To reduce extraneous cognitive load can also be done by considering the number of worked examples presented to students. First, it must be noted that this kind of instruction is suitable for anovel or complex learning material. In other words, it is given for students who have limited prior knowledge. Accordingly, the number of worked example has to be sufficient to students. The degree of complexity has to be considered, including variations of thecontext of the problems [16, 17]. For students who have sufficient prior knowledge, they may not benefit from worked examples. Instead, these may cause redundant information in their working memory and hamper learning [7].

The second principle is to manage intrinsic cognitive load, the complexity of the learning material. Its level of complexity may describe the characteristic of the learning material. The content of a learning material may have anunchanged level of complexity because it characterises this material[6]. However, from the view of prior knowledge possessed by the learner, level of complexity may change according to how much prior knowledge can be retrieved to organise the complexity of the learning material. When dealing with the same learning material, students with high prior knowledge may see it less complex, but those with low prior knowledge may see it very complex. Consequently, creating worked examples should base on how much prior knowledge possessed by the students. Usually, it is suggested to present material successfully from less to more complex. Particularly for students with thelow prior knowledge, the worked example should show correct solutions with clear steps of solution to help them understanding the problem and how to solve it.

The third principle is to increase thegermane cognitive load. When dealing with the interacting elements of learning material, students will experience germane cognitive load that directs students to organise, understand and construct knowledge. By using a worked example based instruction, thegermane cognitive load may be imposed by asking students to apply their metacognitive skills by monitoring their thinking process, doing self-explanation [12], or imagining the context [18]. Students may be asked to write their conclusion or summary of the key concept or procedures they learn from the worked example, or to refer to the key answer. These activities help students to clarify and reflect their knowledge construction and therefore understand more meaningfully.

Further, the format of worked examples should be paired with similar problems [19]. The problem-solving pairs facilitate students to practice and automate the knowledge acquired from the worked example. Technically, the instruction consists of pairs of worked example and a similar problem-solving. The number of pairs depends on the complexity level of each problem solving, and hence instruction designers should consider its intrinsic cognitive load. More importantly, the paired problem solving must have similarity regardingconceptual base or procedural solution steps to the provided worked example.

It has been discussed that teachers have to ensure that the learner fully attends to the worked example tasks during instruction to increase thegermane cognitive load. Chi, Bassok [12] found that most low prior knowledge students often simply look at a glance and did not attempt to study all parts of worked examples fully. Van Merriënboer [20] suggested the use of completion problems to engage students into the worked example. This completion problem requires them to complete some key solution steps in the worked examples by themselves. This strategy could be effective particularly when the problem requires long solution steps. The effectiveness of completion strategy is supported by Paas [21] who investigated the effect of completion problems for learning statistics. Moreover, Paas also found that the completion and worked example conditions required a significantly shorter study time than conventional condition, which is problem-solving instruction.

To explain why partially completing the example is effective, it is argued that students' attention is directed to the problem state and the provided key solution steps while completing the missing solution steps. It may also be said that completion problems are a combination of worked examples and problem-solving, which is an alternative format of worked examples based instruction. In turn, completion problems have been used to develop a strategy of presenting worked examples namely the fading guidance effect.

3. Faded-Example

The guidance fading strategy uses a combination of worked examples, completion problems, and problem-solving which are presented sequentially, and designed to facilitate a smooth transition from novice to more knowledgeable students [22-24]. This strategy is developed based on the expertise reversal effect. Novice students would be more advantaged by anexplicit instruction like the worked examples based instruction, but master students would be more advantaged by animplicit instruction like the problem solving based instruction.

Renkl, Atkinson [23] suggested two fading techniques; these are called backwards and forward fading examples. The *backwardsfading* strategy, the first worked example is fully completed, the second worked example has the solution to the final step removed, the third has the two last steps removed, and so forth, until the final example presents the whole problem-to-be-solved only. The task is to complete the removed steps, whose number increases as knowledge automation develops. For the *forward fading* strategy, the series of completion are provided in the opposite direction, that is the first step of the solution is incomplete, then the second step is incomplete, and so forth, in a forward direction until the full incomplete problem is presented.

Research evidence showed by Renkl, Atkinson [23] indicated that the backwards fading strategy is more favourable for low prior knowledge students because they are benefitted from studying a full worked example at the beginning of the learning phase. Reisslein, Sullivan [25] supports that slow fading strategy was more advantageous for low prior knowledge students transitioning from worked example stage to independent problem-solving. Slow fading strategy uses a backwards fading strategy that provides students with a longer phase of knowledge acquisition. In contrast, a fast fading strategy was found to be more advantageous for high prior knowledge students.

3.1. Developing Faded-Example Instructions

In a problem-solving type that has at least, for instance, four steps to complete the answer, using *backwards fading* strategy means providing the beginning steps as the workedexamples, and the last steps as the task students need to accomplish. The following example shows the use of backwards fading examples when studying how to simplify a trigonometry formula which commonly has the same number of solution steps. Four problems are provided in which the first problem has the fourth step is faded, the second one has the third, and the fourth are faded, the third one has the first step only shown, and the last one has no steps shown (problem-solving). The solution is provided in a table to ensure it is easy to match between the explanation and the execution of the solution steps. Different colours are used to reduce search of information. Also, this series of faded examples have asimilar basic concept to solve the problem that is manipulating the trigonometry form into trigonometry identity or basic trigonometry. A Little modification in a series is suggested to minimise unnecessary cognitive load. Thus students are facilitated more on knowledge acquisition and automation.

Task: Study how to simplify trigonometric forms by completing the problem solution below.

1. Simplify: $sin\ t + cot\ t\ cos\ t$

Answer:

Explanation	Execution
Step 1 Substitute $cot\ t$ with $\left(\frac{cos\ t}{sin\ t}\right)$	$sin\ t + cot\ t\ cos\ t = sin\ t + \left(\frac{cos\ t}{sin\ t}\right) cos\ t$
Step 2 Form into a fraction with denominator *sin t*. By this way, the nominator makes the trigonometry identity $sin^2t + cos^2t$	$\Leftrightarrow$ $sin\ t + cot\ t\ cos\ t = \frac{sin^2t + cos^2t}{sin\ t}$
Step 3 Substitute $sin^2t + cos^2t$ by 1	$\Leftrightarrow$ $sin\ t + cot\ t\ cos\ t = \frac{1}{sin\ t}$
Step 4 Simplify $\frac{1}{sin\ t}$ into $cosec\ t$	$\Leftrightarrow$ $sin\ t + cot\ t\ cos\ t = \ldots$

2. Simplify $cosec\ t - cot\ t\ cos\ t$

Answer

Explanation	Execution
Step 1 Substitute $cosec\ t$ with $\frac{1}{sint}$ and $cot\ t$ with $\left(\frac{cos\ t}{sin\ t}\right)$	$cosec\ t - cot\ t\ cos\ t = \left(\frac{1}{sint}\right) - \frac{cost}{sint}\ cost$
Step 2 Simplify the fraction on the right-handside, and hence the nominator makes trigonometry identity $1 - cos^2t$	$\Leftrightarrow$ $cosec\ t - cot\ t\ cos\ t = \frac{1-cos^2t}{sint}$
Step 3 Substitute $1 - cos^2t$ with sin^2t	$\Leftrightarrow$ $cosec\ t - cot\ t\ cos\ t = \ldots$
Step 4 …	$\Leftrightarrow$ $cosec\ t - cot\ t\ cos\ t = \ldots$

3. Simplify $cos\ t + tan\ t\ sin\ t$

Answer

Explanation	Execution
Step 1 Substitute $tan\ t$ with $\left(\frac{sin\ t}{cos\ t}\right)$	$cos\ t + tan\ t\ sin\ t = cos\ t + \left(\frac{sin\ t}{cos\ t}\right) sin\ t$
Step 2 Simplify the fraction on the right-handside, and hence the nominator makes trigonometry identity $cos^2t + sin^2t$	$\Leftrightarrow$ $cos\ t + tan\ t\ sin\ t = \ldots$
Step 3 …	$\Leftrightarrow$ $cos\ t + tan\ t\ sin\ t = \ldots$
Step 4 …	$\Leftrightarrow$ $cos\ t + tan\ t\ sin\ t = \ldots$

4. $sec\ t - tan\ t\ sin\ t$

Answer

Explanation	Execution
Step 1 Substitute $\tan t$ with $\left(\frac{\sin t}{\cos t}\right)$	$\sec t - \tan t \sin t \ = \dots$
Step 2 …	$\Leftrightarrow \quad \sec t - \tan t \sin t = \dots$
Step 3 …	$\Leftrightarrow \quad \sec t - \tan t \sin t = \dots$
Step 4 …	$\Leftrightarrow \quad \sec t - \tan t \sin t = \dots$

5. Conclusion: to simplify a trigonometry form, manipulate the given form by factorization, substitution, expansion or simplification based on basic trigonometry or trigonometry identity.
6. Create a trigonometry problem similar to the above problems, and then solve it. If it is still too hard, try to solve it again and give anexplanationof each step of thesolution.

The last two tasks above provide students to understand their learning more meaningfully, as discussed previously regarding the germane cognitive load. In a problem-solving type that has at least, for instance, four steps to complete the answer, using *forward fading* strategy means providing the last steps as the workedexamples and the first steps as the task students need to accomplish. In the backwards faded examples above, solution step 4 is faded, then step 3, 2 and 1. In the faded forward examples, the very first instruction contains solution step 1 is firstly faded while solution step 2, 3 and 4 are showed. Then, in the second instruction, step 1 and 2 are faded; in the third instruction, step 1, 2, and 3are faded. It has to be noted that students are instructed to complete the faded example from the first step while studying the pattern of the problem solution.

When aninstructionis focused on facilitating thetransition from fully-guided problem solver into independent problem solver, the faded example strategy can be implemented. It is critical that teachers develop this kind of instruction carefully. Thus, students can manage their cognitive load, either in terms of intrinsic cognitive load (i.e., the arrangement of the level of complexity), extraneous cognitive load (i.e., the arrangement of the presentation of the connected information), and germane cognitive load (i.e., how the task engage students in meaningful learning). It should be noted that creating this instruction requires adeep understanding of the content material as well as how to present problem solution examples that assist students in automating their knowledge more efficiently.

4. Conclusion

Learning is an activity to acquire and automate knowledge. When learning new or complex problem to solve, working memory has to load high demand of cognitive process because of lack of prior knowledge can be retrieved from long-term memory. For this category, students are better facilitated with an explicit instruction such as worked example based instruction. On the other hand, when students can organise learning material efficiently because the already well-possess knowledge base to be applied, students will improve their learning by an implicit instruction, such as discovery learning. When students have acquired some level of prior knowledge but limited in use, teachers could facilitate them with a faded example based instruction using either forward or backwards strategy. This instruction consists of a set of completion problems that require students to filling incomplete solution steps. The solution steps were faded gradually to ensure students attend the task fully and hence acquire and automate their knowledge. Eventually, theinstructional strategy must be developed by considering the cognitive load may be imposed whenstudents use it.

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